2. Birch's law and equation-of-state parameters

Birch [4, 5] in 1961 observed that the velocity of compressional waves V_p in minerals and rocks depends essentially on density in a linear fashion for substances with similar mean atomic weight. He made this important observation on the basis of his systematic investigation [5, 6] of the behavior of compressional wave velocities in minerals and rocks at high pressure. Birch [4] then concluded that the velocity V_p in isotropic aggregates of oxides and silicates is mainly a function of density and mean atomic weight and that the variation of density for substances with the same mean atomic weight reflects structural and compositional differences. This is known today as Birch's law for the velocity-density-mean atomic weight relation. McQueen et al. [7] observed for similar materials that the bulk sound velocity V_ϕ, frequently called the "hydrodynamical velocity", is proportional to density. Wang [8] also observed the linear variation of the bulk sound velocity with density in *compressed* periclase. Based on velocity measurements of shear waves V_s in minerals and rocks, Simmons [9] observed that the shear wave velocity also is a linear function of density, although a few exceptions to the above were noted by Simmons [9]. These observations essentially support Birch's law, and they have important implications in geophysics since they enable one to estimate velocities of elastic waves in solid phases that are as yet unmeasured.

The velocities of compressional and bulk waves in olivines with different Fe/(Mg + Fe) ratios are plotted in figs. 1 and 2, respectively, as a function of density. The data on these velocities are from recent work of this author, and they are based on a systematic measurement of the velocity of both compressional and shear waves in synthetic olivine polycrystals [3, 10]. Under pressure, the density of an olivine with a specific (Fe/Mg) ratio would increase through a gradual decrease in mean atomic volume, while the velocity of elastic waves will increase in the manner illustrated in fig. 3. The path indicated by the dashed line would be the actual velocity-density trajectory as the olivine is compressed. Olivine transforms into a spinel structure at high pressure; this phase change is accompanied by a density increase of 10 to 12%. Knowing the end-point density of the olivine-transformed spinel, we can estimate its velocity in the manner illustrated in fig. 3. Thus, as noted before [3, 10], the velocity

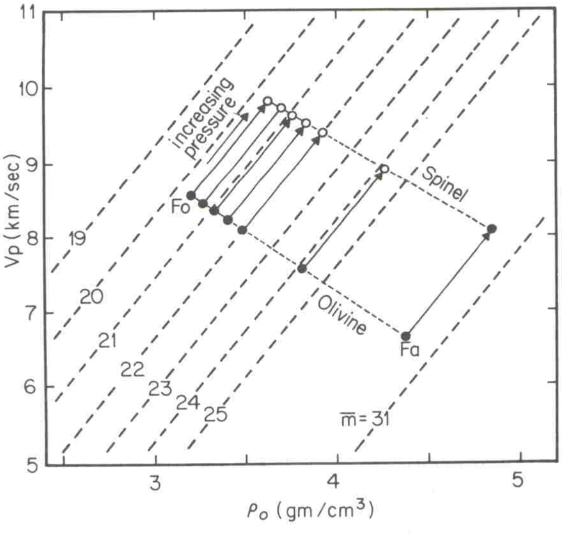

Fig. 1. Velocity of compressional waves-density-mean atomic weight relation for olivine and the olivine-transformed spinels in the $(Mg_xFe_{1-x})_2SiO_4$ system. The olivine data are from refs. [3, 10] and the olivine-transformed spinel data are estimated values.

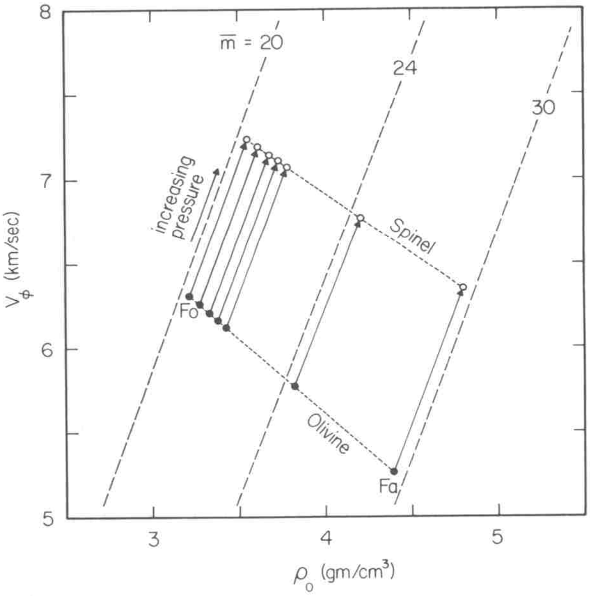

Fig. 2. Bulk sound velocity-density-mean atomic weight relation for olivines and the olivine-transformed spinels in the $(Mg_xFe_{1-x})_2SiO_4$ system. The olivine data are from [3, 10] and the olivine-transformed spinel data are estimated values.

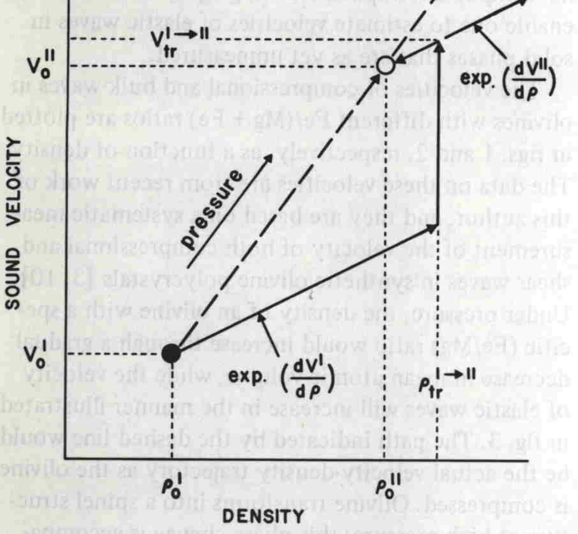

Fig. 3. The density-velocity relation for a solid undergoing the first-order phase change.

after interpolation to zero pressure would be represented by an intercept of two lines drawn from density and mean atomic weight as shown in figs.1 and 2. In table 1 these estimated velocities for the olivine-transformed spinels in the Mg_2SiO_4–Fe_2SiO_4 system are listed. In their recent publication, Mizutani et al. [11] reported that they have successfully prepared a fayalite-transformed spinel sample and measured the compressional velocity of 8.0 (±0. 1) km/sec at ambient conditions. This value is about 21% greater than the compressional velocity measured in fayalite samples as noted in ref. [10]. The compressional velocity of 8.0 km/sec [11] is in remarkable agreement with 8.05 km/sec, a V_p value found by the intercept of the two lines drawn for the density and the mean atomic weight [3, 10]; thus, again, Birch's law is seen to be confirmed.

The bulk modulus also changes with phase change. Since the adiabatic bulk modulus K_s is related to density ρ and the bulk sound velocity V_ϕ by

$$K_s = \rho V_\phi^{\ 2} \qquad (1)$$

K_s can be evaluated. The K_s values thus found for the olivine-transformed spinels are listed in the 8th column of table 1 and compared in fig. 4 with the respective K_s values in the olivine structure. Values of the bulk modulus show about a 45% increase for β–Mg_2SiO_4 (spinel) and 61% increase for the Fe_2SiO_4–spinel from their respective bulk modulus values in the olivine phase.

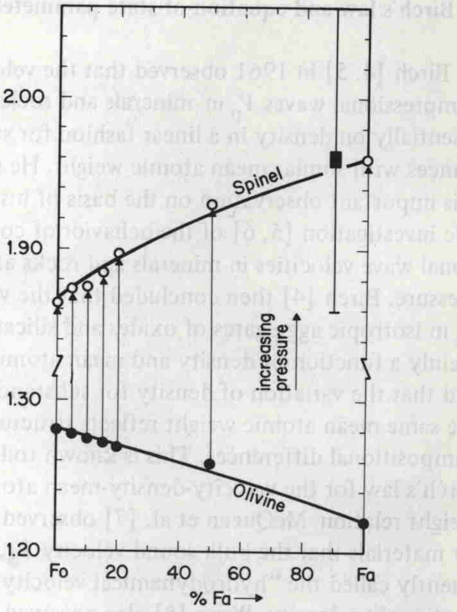

Fig. 4. Bulk modules of olivines and the olivine-transformed spinels in the $(Mg_xFe_{1-x})_2SiO_4$ system. The olivine data points are from [3, 10] and the olivine-transformed spinel data points are estimated values. The datum point (⧯) is from isothermal compression data in ref. [12] found by a curve-fitting procedure using the Birch equation of state with the assumption that $(\partial K/\partial p) = 4$.

These estimated K_s values may be compared with literature data on isothermal bulk modulus values reported by Mao et al. [12], after appropriate correction for adiabatic to isothermal modulus is made. Mao et al.[12] presented values of the isothermal bulk modulus K (found by a curve-fitting procedure using the Birch equation of state with the assumption that $(\partial K/\partial p)_T = 4$) for three olivine-transformed spinels from their measurements of volume change with pressure. Their results [12] are 2.12 (±0.10), 1.96 (±0.10), and 2.08 (±0.10) Mb for the isothermal bulk modulus of Fe_2SiO_4–spinel, $(Mg_{0.1}Fe_{0.9})_2SiO_4$–spinel, and $(Mg_{0.2}Fe_{0.8})_2SiO_4$–spinel, respectively. These values of K reported by Mao et al. [12] compare favorably with 1.96, 1.95, and 1.94 Mb, respectively, calculated from eq. (1) for spinels of the same compositions.